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Research Article

Play based activities for mathematical thinking at infancy: Nursery teachers' and parents' beliefs

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Abstract: Mathematics takes place in a major part of human life and mathematical concepts are used in every part of daily life, starting from the age of infancy. The present study concentrates on the age of 11 months to two years, during nursery education, when formal, informal and non-formal activities enable infants to have experiences related to mathematical concepts. Nursery teachers are expected to include play-based activities at every stage of the teaching process, while parents are recognized as young children's first educators. We examined nursery teachers' and parents' beliefs and practices about the development of the infants' mathematical skills through the use of play-based activities and their respective roles. The present study was conducted in Cyprus, where obligatory preschool education is only one year before primary education. Questionnaires, interviews and shared diaries with home activities were used for quantitative and qualitative data. Results indicated that both groups of participants expressed positive conceptions on the value of daily life play-based activities which could support mathematical learning. However, it seemed that in the case of parents there was a lack of relevant knowledge about the use of attractive and creative activities which could relate to plenty of mathematical concepts. Parents recognize the vital role of teachers and they asked for further guidance and support. We discuss how we can ensure the quality of early mathematics informal teaching and nonformal learning experiences can be offered for all infants. We discuss the role of the Curriculum in Mathematics at nursery school under a play-based context and the guided parental involvement.

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Play-based activities; Teachers' and parents' beliefs; Mathematical thinking

Introduction

The National Council of the Teachers of Mathematics and the National Association for the Education of Young Children underline that high quality, challenging and interesting education at early ages is a vital foundation for future mathematical performance, by concentrating at the ages of 3 - 6 years old (Cerezci, 2020). Most studies concentrated on those ages, as most of the Curricula in Mathematics worldwide are starting from the age of 3 or 5 years old without any reference at the nursery education. The role of nursery education for infants and the role of non-formal learning through parental involvement have not been examined thoroughly. We have to keep in mind that children start playing before they walk or speak. Play allows the children to show their feelings, emotions and ideas (Ozdogan, 2011). There are plenty of studies on the effect of play-based activities on language learning, cognitive development and mathematical skills (Derman et al., 2020). However, only a few of them are referred to preschool education and mainly nursery education.

The present study concentrates on the age of 11 months to two years old, during nursery education. Nursery teachers are expected to develop infants' mathematics competencies by including play-based activities. They need to create and support an appropriate pedagogical environment and at the same time, they are expected to cooperate with parents in order to guide them on how they could have productive learning time with their infants. We have to underline that we are talking about educational systems where

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the availability of universal childhood education has become a reality and we now need further to ensure that even at the ages of non-obligatory education the teaching and learning experiences are qualitative for all children. The present study was conducted in the educational system of Cyprus, where obligatory education started one year before primary education. Most children attend a private preschool starting at the age of 2 or 3 years. Infants are either under the care of their grandparents or attend private nursery schools. Those nursery schools are under the supervision of the Ministry of Labour and not the Ministry of Education and it does not have to base their program on a Curriculum.

The present study is a part of a project about the development of mathematical thinking at the age of infancy through the use of formal and informal play-based activities at school and at home. The aims which are presented here concentrated on (a) the nursery teachers' beliefs and practices about the development of mathematical skills at early ages and the respective role of play-based activities and (b) the parents' beliefs about their own role through the everyday activities with their children and their beliefs about the respective nursery teachers' role.

Nursery Education and Research on Mathematics Learning at the Age of Infancy

Contemporary research on early mathematics education focuses on children from birth until they enter formal schooling (Bjorklund et al., 2020). The starting age of formal schooling is different for each educational system. In some cases, the formal Curriculum is starting at the age of 5 years old and in a few cases at the age of 3 years old without having any reference to the previous ages. Cognitive psychology and research on early ages indicated that infancy and the first childhood are the periods where the fastest development is observed (Demetriou et al., 2020). Anthony and Walshaw (2009) argue that the development of mathematical competencies begins at birth and there are enriched through everyday experiences. The structured experiences offer added value to the learning processes. At the same time, the period from birth to the start of school is an important developmental phase where children acquire fundamental knowledge and develop basic skills necessary for later learning and school achievements (Soto–Calvo & Sanchez-Barrioluengo, 2011). Even the time spent in childhood education and care centres seemed to predict the child's numeracy competencies in future (Anders et al., 2013) or at least in the first grades of primary education (Aunio & Niemivitra, 2010). In order to face the inter-individual differences among children in their mathematical abilities at school entry, we need to study further the role and impact of nursery education in relation to parental involvement.

Ideally, the learning of mathematics should begin since birth and continue always as children explore the world around them by themselves. In order to do that they need important figures such as their parents and the nursery teachers in order to assist them positively and productively to construct and enrich their cognitive structures (Irma et al., 2017). In this way, they could construct new knowledge and use their cognitive abilities fluently and flexibly. Parviainen (2019) argues that the mathematical skills gained in early childhood influence later mathematical achievement and consequently it is clear the importance of strengthening mathematics learning and skills in early childhood education (Sarama & Clements, 2009). Watts et al. (2014) indicated that the mathematics competencies children demonstrate at school entry are the strongest predictors of their later school achievement. In order to talk about teaching and learning mathematics in the early years, we have to conduct studies in various complex, multifaceted and dynamic learning environments which include at least home activities and nursery school activities. Everyday activities provide the stimulus for informal mathematical development. For example, young children at the age of infancy can learn about patterns through rhymes and songs (Anthony & Walshaw, 2009).

The majority of the children when they start kindergarten at the age of 3 years old, can count small sets of objects and they share objects equally into groups. Earlier studies identified the phenomenon of subitizing, according to which children are able to visually process objects of four or fewer objects without counting them (Bruce et al., 2016). All the Curriculum in mathematics included at least five main thematical units: numbers/operations, geometry, measurement, patterns/relations and statistics/probabilities. Many times, there is a misconception and an identification of numeracy with mathematics learning and for this reason, research concentrated on number development and numeration (Yilmaz, 2017). Elia (2018)

concentrated on geometric and spatial thinking in early childhood by emphasizing the role of the body and other semiotic resources. When babies locate themselves and objects in space they are using either landmarks or geometric cues (Cross et al., 2009). Several studies indicated that children gradually learn to build mental images of the surrounding environment and create mental maps for navigation by using spatial abilities (Clements & Samara, 2007). Undoubtedly the mathematical concepts are interrelated and when children learn measurement, they connect length with numbers (Hawes et al., 2017).

The Play-Based Learning at the Age of Infancy and Nursery Teachers' and Parents' Beliefs

Play is the leading activity that children enjoy. They start to play before they walk or speak (Ozdogan, 2011) and consequently it is the basement of the social, physical, emotional, mental and cognitive development of the child. Through play, young children and infants develop their self-confidence and construct their self-image. The play has long been regarded as a critical element of early childhood curriculum and pedagogy (Dockett & Perry, 2010). Defining play and play-based learning is not straightforward (Carolan et al., 2020). Much of what we know about play and its relationship with learning has its roots in theorists such as Piaget, Vygotsky and Bandura, who underline the role of play, investigation and exploration, the role of language and communication. There are numerous of studies on the effect of play-based mathematical activities on mathematical skills, language and cognitive development at the ages of preschool education (Derman et al., 2020).

One of the main aims of mathematics education is to ensure that mathematical learning is meaningful and enjoyable for children. The Curriculum, the teaching processes and the learning environment need to retain a sense of playfulness in order to ensure the fun and positive dispositions through the experiences (Lee, 2010). Similarly, the everyday activities at home can be used as part of informal or non-formal learning. For example, the activities of sharing, cooking, completing puzzles and estimating provide opportunities for young children to practice and develop mathematical competencies.

As Bruce et al. (2016) underline, while children demonstrate remarkable abilities with many mathematical concepts, many nursery teachers do not have positive beliefs about the necessity to "teach" mathematics or they do not have the necessary competence and confidence on engaging meaningfully with both the children and the content of mathematics. Their knowledge, their beliefs and their self-efficacy beliefs are the most important factors that influence their practices in the school environment. The absence of a formal curriculum at these ages does not enable teachers to depend their decision on a theoretical framework derived from experts and policymaker stakeholders.

The second vital dimension is parents. Parents are recognized as young children's first educators. Studies about the home numeracy environment have in most cases found a positive relation between indices of the home numeracy environment with the children's number skills (Soto-Calvo , 2019). However, their impact differs based on their socioeconomic background (Nguyen et al., 2016). For example, Levine et al. (2010) indicated that parents with low background provide more input about simple counting than parents with a high background who emphasize more estimation, number cardinality and in general advanced sense skills.

Levine et al. (2010) studied for many years in a longitudinal study on the role of parental math talk on the children's acquisition of cardinality. It was impressive that children by the age of 30 months old were able to count and label cardinal value sets. Similarly, Casey et al. (2008) used block–building interventions in relation to storytelling and they indicated that storytelling provided an effective context for teaching geometrical concepts and spatial sense. Carolan et al. (2020) examined families' experiences of a funded play-based early learning program for children in the year before school entry. Results indicated mainly the importance of good communication with teachers in order to understand the play-based framework.

Sometimes parents try to use typical school-based activities through textbooks in order to accelerate the development of their children's knowledge and skills. However, they need to relate the intended goals with play; otherwise, they create to their children a disposition of boringness towards school. They have to

know the balance in order to promote both play and mathematical understanding (Dockett & Perry, 2010). The learning opportunities need to arise from both naturally occurring informal experiences and from planned activities (Anthony & Walshaw, 2009) that are play-based.

The present study focuses on how nursery teachers construct beliefs and self-efficacy beliefs about their role to be engaged in both play-based pedagogy and mathematics as a curriculum discipline. The main challenge is for an integrated mathematical curriculum for very young learners in order to develop mathematical thinking through a bridge that connects in a realistic and natural context the pedagogy, with the curriculum and the related learning communities at school and home. The role of activities at home through non-formal learning processes engages parents whose beliefs guide their actions.

Method

In order to examine teachers' and parents' beliefs about the role of play-based activities on the development of infants' mathematical thinking at the age of 11 months to 2 years old, a mixed methodology was used. We aimed to relate teachers' and parents' beliefs with their practices at school and their everyday home activities respectively. The quantitative part is based on data collected through a questionnaire for the nursery teachers' beliefs and a questionnaire for the parents of a nursery school which was examined as a case study, due to the first researcher's special relation which increased participants' confidence. The qualitative part is based on semi-structured interviews with a group of parents and the activities they kept as a diary and shared them with us.

Sample

The sample for the quantitative part of the study was consisted of two main groups of participants: nursery teachers and infants' parents. 110 nursery teachers who were working with children up to the age of 2 years old and 99 parents (44 fathers and 55 mothers) whose children went to a nursery school took part at the study. The sample was not representative as due to the pandemic of Covid-19 we were not able to visit the nursery schools. We had used social media for finding the sample of the nursery teachers, by sharing the online questionnaire and having it open for 3 days. Obviously, the sample was not representative and probably participants who took part voluntarily had specific characteristics which were not examined by this study. We had visited a nursery school in Cyprus, where the second researcher had easy access due to personal relations, in order to ask parents to take part in the study. With the contribution of the director, almost one parent of each child of all the children of the school (99 parents of 105) completed the questionnaire. A small group of 7 parents took part in individual semi-structured interviews and kept a diary for a week where they were asked to present the home activities with their children which were related to the development of mathematical thinking. The sample of a small group of parents accepted to spend a few more time in order to take photos of the activities with their children and describe them. Although we knew that probably the parents who accepted voluntarily to take part in the second part of the study had extra interest and they did not represent even all the parents of the school we aimed to have indications about the activities they used. All the participants were informed that they could abort from participation in the study and the parents of the qualitative part of the study gave their signed permission to use the photos without revealing their children's identities.

Measurement Tools

A questionnaire was constructed for the measurement of nursery teachers' beliefs and a questionnaire for the measurement of parents' beliefs. The teachers' questionnaire consisted of 28 Likert type items (1= strongly disagree and 5 = strongly agree). The items are presented in Table 1, as part of the results of the exploratory factor analysis. Items were about their beliefs about the epistemology of learning mathematics, their beliefs about the teaching of mathematics in nursery and preschool education, the curriculum of mathematics and the teaching processes which mainly reveals the role of play. The parents' questionnaire consisted of 31 items (Table 2) about the development of mathematical thinking, the role of play in their children's life, their beliefs about the role of the nursery teachers and their role as the constructors of their children's everyday activities at home. A protocol was used for the individual semi-

structured interviews with parents and guidelines were given to them in order to keep a weekly diary with the activities they organize for their kids and there were related to mathematics.

Data Analysis

Quantitative data were analyzed by using the SPSS. Exploratory factor analysis was used for each one of the questionnaires in order to identify the major factors which constructed nursery teachers' and parents' beliefs. After the content analysis of the factors, we had conducted descriptive statistics on them. Qualitative analysis was used in order to examine specific aspects of parents' beliefs and mainly in order to identify the play-based activities they organize for their infants which were related to the development of mathematical thinking. We aimed to relate their actions with their respective beliefs. Specifically, we had used content analysis of the interviews in order to concentrate our attention on the activities they used, the games they buy or organize, their expectations about the teachers' role and their beliefs about their role. The diaries were used in order to identify activities that could confirm their practices whether there were previously expressed or not.

Results

We first subjected the nursery teachers' responses to exploratory factor analysis in order to examine the extent to which the questionnaire statements reflected the main dimensions under examination. The analysis of the participant's responses to the items resulted in four factors with eigenvalues greater than 1 (KMO=0.807, $p<0.05$). Those four factors explained the 68.295% of the total variance. The first factor which was consisted of 7 items reflected on nursery teachers' beliefs about the infants' understanding of mathematical concepts. The second factor was consisted of 7 items about the development of children's mathematical thinking. The third factor was about the role of play on the infants' mathematical understanding and the fourth factor was about the role of play on the children's mathematical understanding. At Table 1 the loadings of the items on the factors are presented.

The means (min=1 and max=5) of the four factors were high (Table 2) indicating the positive beliefs that nursery teachers have about the role of teaching on the development of mathematical thinking at an early age and mainly the significant role of the play-based activities. They recognize the predominant role of their work as the organizers of the activities at school and as the experts who have to guide the parents for relevant informal activities in authentic framework.

Table 1. The factor loadings of the items of the nursery teachers' questionnaire

Statements	Factors			
	1	2	3	4
It is necessary to master the subject of mathematics in order to be able to teach mathematics to all children.	.942			
Learning maths requires talent.	.916			
Every child can learn mathematics.	.874			
Mathematical teaching through play facilitates the learning and acquisition of mathematical concepts by all children.	.866			
Using play for teaching mathematical concepts to children in infancy is difficult for me.	.867			
My classroom's infants enjoy using mathematical play.	.785			
My classroom's infants enjoy participating to mathematical activities.	.418			
I organise mathematical teaching activities through play		.855		
I organise mathematical teaching activities through fairy tales.		.813		
I organise mathematical teaching activities through outdoor play.		.722		
During meeting with parents, we discuss about activities they can do with their children.		.784		
During meeting with parents, we discuss about ways they can take advantage of playing with their children.		.531		
During meeting with parents, I suggest them board games that are useful to play with their children.		.651		
During meeting with parents, I suggest them exploration activities they can have with their children in the countryside, in the yard, in a park.		.755		

Play contributes positively to the infants' acquisition of the mathematical concept of "numbers" (0-10).	.737	
Play contributes positively to the infants' acquisition of the mathematical concept of "spatial orientation" (up/down/inside/outside).	.659	
Play contributes positively to the infants' acquisition of the geometrial concepts	.728	
Play contributes positively to the infants' acquisition of the mathematical concept of "measurement" (small / medium/ large / little/ very).	.601	
Play contributes positively to the infants' acquisition of the mathematical concept of "patterns".	.822	
Play contributes positively to the infants' acquisition of the statistical concepts.	.813	
Play contributes positively to the children's acquisition of the mathematical concept of "numbers" (0-10).		.815
Play contributes positively to the children's acquisition of the mathematical concept of "spatial orientation" (up/down/inside/outside).		.908
Play contributes positively to the children's acquisition of the geometrial concepts		.882
Play contributes positively to the children's acquisition of the mathematical concept of "measurement" (small / medium/ large / little/ very).		.898
Play contributes positively to the children's acquisition of the mathematical concept of "patterns".		.880
Play contributes positively to the children's acquisition of the statistical concepts.		.743

Table 2. Means and SDs of the four factors which derived by nursery teachers' questionnaire

Factors	Mean	Standard Deviation
F1: Infants' understanding of mathematical concepts	3.33	.505
F2: Development of children' s mathematical thinking	3.33	.440
F3: The role of play on children's mathematical thinking	3.44	.520
F4: The role of play on children's mathematical understanding	3.47	.484

The analysis of the parents' responses at the items resulted in six factors with eigenvalues greater than 1 (KMO=0.736, $p<0.05$). Those six factors explained the 75.401% of the total variance. The content analysis of the items resulted in the following up factors: F1: Beliefs about the children's mathematical thinking development, F2: Beliefs about the acquisition of the mathematical concepts by their children in relation to their attitudes towards mathematics, F3: Beliefs about the ways of developing children's mathematical thinking, F4: The role of nursery teachers on the development of children's mathematical thinking, F5: The contribution of play to the acquisition of mathematical concepts by infants and F6: The contribution of play to the acquisition of mathematical concepts by children. In Table 3 the loadings of the items on the factors are presented.

The means of the six factors were high (Table 4) with only the exception of the second factor (1.70). However, the second factor underlined the role of parents to facilitate their children to take part in activities that engage in mathematics although their own negative experiences led them to have negative attitudes towards mathematics and probably low self-efficacy beliefs. The results underlined their expectations of nursery teachers to use play-based activities for the development of infants' and children's mathematical thinking and understanding of mathematical concepts.

Table 3. The factor loadings of the items of the nursery teachers' questionnaire

Statements	Factors					
	1	2	3	4	5	6

It is necessary for all the children to understand the mathematical concepts	.597					
All children can learn mathematics.	.767					
Every child can learn mathematics through play.	.748					
Teaching mathematics at nursery education have to include the use of playful activities.		.706				
It is necessary for me as parent to like mathematics in order to engage my child in mathematical activities.		.794				
It is necessary for me to like mathematics in order to engage my child in mathematical games at home.		.477				
I enjoy using toys at home in order to "teach" mathematics to my child.			.836			
It is easier for me to use play in order to "teach" my child maths at home.			.882			
I use fairy tales in order to "teach mathematics to my child.			.810			
I teach mathematics to my child through our play inside and outside the home			.582			
Nursery teachers need to use the play in order to enable my child to understand various mathematical concepts.				.438		
Nursery teachers need to organize structured play-based activities on a daily basis in order to enable my child to construct mathematical concepts.				.894		
Nursery teachers need to provide the necessary feedback to my child when he / she performs mathematically organized playful activity in a wrong way.				.918		
The role of kindergarten teachers during playful mathematical activities should be active and supportive for my child.				.849		
Play contributes positively to the infants' acquisition of the mathematical concept of "numbers" (0-10).					.925	
Play contributes positively to the infants' acquisition of the mathematical concept of "spatial orientation" (up/down/inside/outside).					.890	
Play contributes positively to the infants' acquisition of the geometrial concepts					.909	
Play contributes positively to the infants' acquisition of the mathematical concept of "measurement" (small / medium/ large / little/ very).					.895	
Play contributes positively to the infants' acquisition of the mathematical concept of "patterns".					.882	
Play contributes positively to the infants' acquisition of the statistical concepts.					.739	
Play contributes positively to the children's acquisition of the mathematical concept of "numbers" (0-10).						.939
Play contributes positively to the children's acquisition of the mathematical concept of "spatial orientation" (up/down/inside/outside).						.907
Play contributes positively to the children's acquisition of the geometrial concepts						.901
Play contributes positively to the children's acquisition of the mathematical concept of "measurement" (small / medium/ large / little/ very).						.905
Play contributes positively to the children's acquisition of the mathematical concept of "patterns".						.882
Play contributes positively to the children's acquisition of the statistical concepts.						.764

Table 4. Means and SDs of the Six Factors which Derived of Parents' Questionnaire

Factors	Mean	Standard Deviation
F1: Beliefs about the children's mathematical thinking development	3.61	.373
F2: Beliefs about the acquisition of the mathematical concepts in relation to their attitudes towards mathematics	1.70	.442
F3 :Beliefs about the ways of developing children's mathematical thinking	3.49	.421
F4: The role of nursery teachers on the development of children's mathematical thinking	3.50	.417
F5 : The contribution of play to the acquisition of mathematical concepts by infants	3.72	.408
F6: The contribution of play to the acquisition of mathematical concepts by children	3.54	.459

During the interviews, parents expressed their beliefs that authentic activities which engage

mathematical concepts contribute positively to the enhancement of their children's mathematical performance in future. A father told us that during the age of nursery education "teacher offers to kids the first experiences with mathematical concepts which will understand in future", while "parents try to enhance their children's probabilities for future success. The knowledge of mathematics is necessary for the success". The examples they proposed concentrated mainly on number acquisition, numerosity and the representation of symbols. They acknowledged the importance of all the mathematical concepts (with references to measurement, patterns, statistics); however, they admitted that they did not have the knowledge and the skills to use appropriate play-based activities. A mother claimed that she "tried to understand the content and the context of the activities at nursery school in order to repeat them at home. I have friends who are related to education and I asked them to suggest to me the most appropriate games in order to buy them for my kids. I do not feel comfortable asking the nursery teacher". Similarly, a father said that he tried to buy a few of the games that were used at the school as he did not know himself what was appropriate. All the parents seemed to ask for further communication with the nursery teachers in order to guide them to understand what types of games they could buy and what types of activities they could organize. They expressed the conception that their children have to "play" at this age and not to be engaged in typical or formal mathematical activities which could create negative dispositions about mathematics. "My older daughter finds mathematics as a boring subject with many symbols. I tried to follow the nursery teachers' guidelines". Two parents said that they tried to relate everyday activities with the learning of mathematical concepts. One of them use "hide and seek" as an example of a simple play that enabled children to learn the multiples of 5. Another one said that she asked her daughter to count objects (dolls, cars, blocks) as part of the play. All the examples of activities that they presented were related to counting and numerosity.

On the contrary, very interesting and impressive were the activities which were included by the parents who accepted to keep and share with us a diary with the everyday activities they had with their kids for a week. The presented parents' activities could be divided into two main types: a) those which depended on games they bought for their infants by having in mind that there were expected to contribute to their development of thinking (Figures 1, 2 and 3) and b) those which depended on everyday activities which were organized purposely by themselves in order to activate their children's mathematical thinking (Figures 4 and 5). In Figure 4, a mother asked her son to spread together with the clothes and they counted together with the clothespins they had used. In Figure 5, a grandmother found her grandaughter to put the snails she found in the yard in a queue and she counted them. Additionally, as they mentioned there were activities that were organized with the contribution of their older children, as "the infants' communication with brothers and sisters acted as a positive example of imitation. For example, a mother asked his older son to hide animal toys in a bowl of flour and her infant tried to find them. Both of them count the animals (Figure 6). The parents who kept the diary were asked to present us with different types of activities and they actually concentrated on numbers (Figure 7) and shapes (Figures 8 and 9). As it is obvious, even in the case of shapes, they were not able to present any activity organized by them.

At the specific nursery school, as parents mentioned, they were informed by the teachers very regularly about their infants' behaviour, performance and activities in different subjects, with emphasis on the learning of the Greek Language and Mathematics. A mother was excited with her infant's teacher as she understood that she organized interesting play-based mathematical activities: "My son acquired many mathematical concepts this year, and I recognize that the success is that this happened through play. Everyday when I asked him to describe his day, he told me about games, tales and creative activities. I teach chemistry in secondary education and I would be happy to find ways to enable children to construct concepts through everyday activities".

Three of the parents underlined their low self-efficacy beliefs in choosing the relevant activities for their infants as they did not have adequate knowledge. They believed that "nursery teachers could use plenty of ways in order to inform them further about their role and how to choose creative, interesting and attractive activities for their children". They wanted to "invest" in their infants' "future success in education through the qualitative time they spend together".


Figure 1


Figure 2

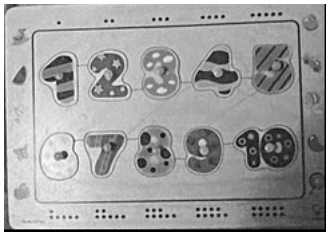
Figure 3


Figure 4


Figure 5


Figure 6

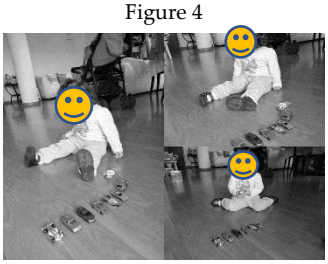
Figure 7


Figure 8


Figure 9

Figures 1-9. Indicative examples of infants' play-based activities

Conclusions and Discussion

Many previous studies have shown that several mathematical skills develop gradually and simultaneously at an early age (Parviainen, 2019). The present study confirmed that nursery teachers and parents believe in their vital role of organizing all the relevant activities in order to construct a rich learning environment for children. Both of them need to ensure that the learning environment is meaningful and enjoyable for children with a sense of playfulness and fun (Lee, 2010). Learning could be enjoyable through play experiences. As Dockett and Perry (2010) underline play has long been regarded as a critical element of the early childhood curriculum. However, the lack of Curriculum at the ages of nursery education seems to lead to limited experiences on how to use play-based activities for plenty of mathematical concepts. Nursery teachers who participated in the study believed in the use of play-based activities for understanding and teaching mathematical concepts related to all the mathematical units. We had not examined their practices; however, parents who seemed to express what they learned through their communication with teachers, used examples that included only numbers and shapes. They were not able to present examples that included measuring quantities (with a weight scale or bowl), repeating patterns at songs, tales or objects etc.

In early childhood, infants need opportunities to learn from both naturally occurring informal experiences and structured activities (Anthony & Walshaw, 2009). Informal teaching and nonformal learning are parts of everyday activities in the authentic environment. Especially parents need guidance and training on how their involvement can be productive and creative through play-based activities which offer fun, creativity, communication and indirectly learning. The present study indicated that they tried to imitate the activities which are organized by the nursery teachers and they would be grateful to have their guidance and support. Having this in mind, nursery teachers can use everyday activities of sharing, cooking, completing puzzles, counting, estimating distances, producing musical rhythms and constructing patterns at school and as suggestions for home activities.

Our results indicate the nursery teachers' positive disposition about the use of play and the necessity to teach mathematics in early childhood through interesting activities for the kids. However, there is a call

for further work by both researchers in the domain of mathematics and nursery education and teachers in the field of early childhood education in order to produce a policy for the development of a Curriculum in Mathematics and similarly in all other disciplines. As Hachey (2013) underlines, after the increased recognition of the importance of early mathematics for later academic success, early childhood mathematics education is now a national (for the United States), and we added international at the same time, priority. We believe that the quality of the nursery school mathematical activities could be improved by constructing a Curriculum for the specific ages. The policy of a play-based process under an interdisciplinary framework has to be the main characteristic of the Curriculum which will be a helpful guide for nursery teachers for their work at school and their suggestions for parents. A Curriculum poses the purpose of an educational system, the objectives, the teaching philosophy, the expected outcomes and the teaching practices. By this way, we will enable the policymakers who are responsible for the curriculum development to pose the goals and the researchers to suggest teaching methods and tools in order to use them for all students by respecting the cognitive, social and inter-individual differences. Undoubtedly the development of a policy does not guarantee the implementation of a relevant and appropriate practice. Aubrey and Durmaz (2012) examined the relationship between policy and practice in the early years' mathematics Curriculum and they indicated that teachers did not implement policy expectations as they brought their own values and understanding to practices. However, the development of a Curriculum with the respective policy is at least the first necessary step.

The present study, as a part of a project which relates the nursery teachers' beliefs and actions with the parental involvement in infants' learning, is a starting point for examining further the establishment of a framework for the teaching and learning processes that can influence positively the mathematics education during childhood ages. Undoubtedly the present study has plenty of limitations which can be the main guide for the development of future studies on the same domain: (a) The sample of the nursery teachers was not representative as we had used social media for sharing the questionnaire, (b) We had examined their beliefs which had to be examined in relation to their real actions at the school environment, (c) The sample of the parents was not representative. Especially the group of parents who had accepted to share their activities with us, they were probably parents with an extra interest in the domain, who had the time to organize many activities for their infants and who had been guided "unconsciously" by knowing the aim of the study to present activities which were related with the development of mathematical thinking (Panaoura, 2021). A future study could examine nursery teachers' knowledge, skills and practices to introduce mathematical concepts of a different domain at the age of infancy. An intervention program could examine the use of alternative training methods in order to explain to parents their role as facilitators in the learning processes of their infants.

Declarations

Authors' Declarations

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